02-711: Computational Molecular Biology and Genomics
Quiz # 1

January 24, 2009

Name:

You have 13 minutes to complete the quiz. The quiz is closed book. You may use a calculator to do numerical computations. If there are any questions, clarifications, or errors, feel free to talk to the instructor or TA. Please make sure you write your name on the quiz.

1 Bayes' Rule [5 pts]

HIV is the virus that causes AIDS. One particular test for HIV is ELISA. To determine the accuracy of ELISA 10,000 people who were known to be HIV positive were identified. This was done using the Western Blot, which is the gold standard test for HIV. These people were then tested with ELISA and 9990 tested positive. Furthermore, 10,000 people who had a negative Western Blot for HIV infection were also identified, and 9980 of these people tested negative using the ELISA test. Suppose in a particular population 1 out of 20,000 individuals are HIV positive, what is the probability of a patient having HIV given he/she was tested positive for ELISA? In other word, what is $P(\text{HIV} = \text{true}|\text{ELISA} = \text{positive})$?

$$
\begin{aligned}
P(\text{HIV} = \text{true}|\text{ELISA} = \text{positive}) &= \frac{P(\text{ELISA} = \text{positive}|\text{HIV} = \text{true})P(\text{HIV} = \text{true})}{P(\text{ELISA} = \text{positive})} \\
&= \frac{(9990/10000)(1/20000)}{(9990/10000)(1/20000) + (20/10000)(19999/20000)} \\
&= 0.0244
\end{aligned}
$$

2 Monty Hall Problem [5 pts]

Suppose you're on the *Let's Make a Deal* game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. As a contestant, you obviously want to maximize your chance of winning the car. Since you are a Bayesian believer (as we just had our first class), we will solve it the Bayesian way. Let C_i be the event in which the car is behind door i for $i = 1, 2, 3$, H_{ij} be the event host opens door j after the player has picked door i, for $i, j = 1, 2, 3$, we want to compute $P(C_1|H_{13})$ and $P(C_2|H_{13})$ and pick the door with the highest probability. What is $P(C_1|H_{13})$ and $P(C_2|H_{13})$? Please show your work.

$$
\begin{aligned}
P(C_1|H_{13}) &= \frac{P(H_{13}|C_1)P(C_1)}{P(H_{13})} \\
&= \frac{P(H_{13}|C_1)P(C_1)}{P(H_{13}|C_1)P(C_1) + P(H_{13}|C_2)P(C_2) + P(H_{13}|C_3)P(C_3)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(1/2)(1/3)}{(1/2)(1/3) + (1)(1/3) + (0)(1/3)} \\
&= \frac{1}{3}
\end{aligned}
$$

$$
\begin{aligned}
P(C_2|H_{13}) &= \frac{P(H_{13}|C_2)P(C_2)}{P(H_{13})} \\
&= \frac{P(H_{13}|C_2)P(C_2)}{P(H_{13}|C_1)P(C_1) + P(H_{13}|C_2)P(C_2) + P(H_{13}|C_3)P(C_3)} \\
&= \frac{(1)(1/3)}{(1/2)(1/3) + (1)(1/3) + (0)(1/3)} \\
&= \frac{2}{3}
\end{aligned}
$$

3 Naive Bayes [5 pts]

Suppose you wake up one cold winter night and find yourself coughing and having a headache but no running nose. You are not sure whether you are having a fever so you decided to build a Naive Bayes model to see the likelihood of you having a fever. Given the Naive Bayes model you build in Figure 1 with its conditional probability distributions shown in Figure 2, what is the likelihood ratio of having fever? In other word, compute

$$\frac{P(F = True|R = False, H = True, C = True)}{P(F = False|R = False, H = True, C = True)}$$

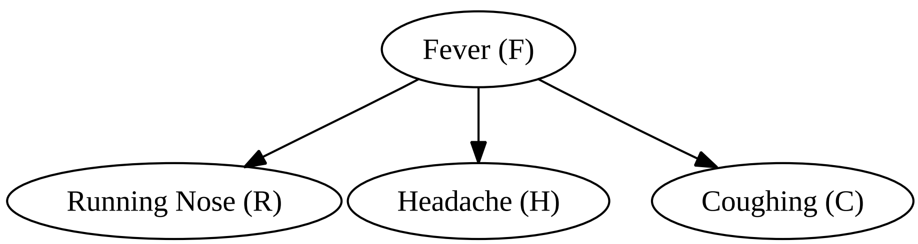

Figure 1: Naive Bayes for diagnosing fever

$P(F = True)$	$P(F = False)$
0.1	0.9

(a) $P(F)$

Fever	$P(R = True\|F)$	$P(R = False\|F)$
True	0.6	0.4
False	0.1	0.9

(b) $P(R|F)$

Fever	$P(H = True\|F)$	$P(H = False\|F)$
True	0.8	0.2
False	0.3	0.7

(c) $P(H|F)$

Fever	$P(C = True\|F)$	$P(C = False\|F)$
True	0.5	0.5
False	0.2	0.8

(d) $P(R|F)$

Figure 2: CPDs for diagnosing fever

$$\frac{P(F = t|R = f, H = t, C = t)}{P(F = f|R = f, H = t, C = t)} = \frac{P(R = f|F = t)P(H = t|F = t)P(C = t|F = t)P(F = t)}{P(R = f|F = f)P(H = t|F = f)P(C = t|F = f)P(F = f)}$$

$$
\begin{aligned}
&= \frac{(0.4)(0.8)(0.5)(0.1)}{(0.9)(0.3)(0.2)(0.9)} \\
&= \frac{0.0160}{0.486} \\
&= 0.3292
\end{aligned}
$$